

STAGE M1 WITH PARTOUT (LIX) (APRIL-JUNE)

## Proof Search in BV

Samar Rahmouni  
samar.rahmouni@polytechnique.edu

Advisor: Dr. Lutz Straßburger  
lutz@lix.polytechnique.fr

BV is a logical system that extends linear logic with a self-dual non-commutative connective[1]. This logic is intrinsically connected to pomset logic. Both were discovered through an interpretation of linear logic as coherence spaces[2]. Coherence spaces introduce a binary noncommutative connective  $\triangleleft$  that carries interesting properties. While both BV and pomset co-exist in the study of coherence spaces, they *are not the same*[3]. Pomset derives proof net syntax that enjoys cut-elimination -but no deductive system, while BV triggered the development of the calculus of structures[1]. This development was a response to the difficulty of finding a sequent calculus that combines commutative and non-commutative connectives. Thus, *deep inference* was found necessary to have a cut-free BV.

While BV and pomset logic enjoy several applications, especially in the field of linguistics, several theoretical questions remain open. Would it be possible to design a deductive system for pomset logic, while skipping the translation to proof nets? Alternatively, can we design a system of proof nets for BV? Furthermore, as it has been shown that all theorems of BV hold in pomset but not the other way around, could there exist a hierarchy of logics with different complexities that also would arise from this same study of coherence spaces[3]. Overall, those questions and the applications of BV are what pushes us to investigate proof search in BV and its implementation. Such implementation would prove useful for future investigations.

The main objective for this internship will be understanding proof search in deep inference proof systems. Previous work was done using *Maude* as a platform[4]. This work demonstrates the design and implementation on classical logic, which we could further extend to linear logic. When it comes to the deep inference needed for BV, the calculus of structures admits some non-determinism that would need to be minimized in order to keep a small search space[5]. Finally, considering automated proof search, we investigate the focused calculus of structures[6] and its possible extension to logics where sequent calculus is inapplicable: BV in this case. If time permits, a user interface for our proof search will be implemented to allow better usage of the tool.

## References

- [1] Alessio Guglielmi. A system of interaction and structure. *ACM Trans. Comput. Logic*, 8(1):1–es, jan 2007.
- [2] Christian Retoré. *Pomset Logic*, pages 299–345. Springer International Publishing, Cham, 2021.
- [3] Lê Thành Dũng Nguyễn and Lutz Straßburger. A system of interaction and structure iii: The complexity of bv and pomset logic, 2023.
- [4] Ozan Kahramanoğulları. Maude as a platform for designing and implementing deep inference systems. *Electronic Notes in Theoretical Computer Science*, 219:35–50, 2008. Proceedings of the Eighth International Workshop on Rule Based Programming (RULE 2007).
- [5] Ozan Kahramanogullari. Reducing nondeterminism in the calculus of structures. pages 272–286, 10 2006.
- [6] Kaustuv Chaudhuri, Nicolas Guenot, and Lutz Straßburger. The focused calculus of structures. In *Annual Conference for Computer Science Logic*, 2011.